CLEARINGHOUSE FOR FEDERAL SCIENTIFIC AND TECHNICAL INFORMATION, CFSTI
DOCUMENT MANAGEMENT BRANCH 410.11

LIMITATIONS IN REPRODUCTION QUALITY

Accession #

☑ 1. We regret that legibility of this document is in part unsatisfactor Reproduction has been made from best available copy.

☐ 2. A portion of the original document contains fine detail which may make reading of photocopy difficult.

☐ 3. The original document contains color, but distribution copies are available in black-and-white reproduction only.

☐ 4. The original distribution copies contain color which will be shown in black-and-white when it is necessary to reprint.

☐ 5.

TS-L-107-9/64 Processor........

THOUGHTS ON LINEAR PROGRAMMING AND AUTOMATION

By

George B. Dantzig

P-824

2 March 1956

The RAND Corporation

1700 MAIN ST. • SANTA MONICA • CALIFORNIA

THOUGHTS ON LINEAR PROGRAMMING AND AUTOMATION*

By
George B. Dantzig

We have all become increasingly aware of the postwar trend towards automation, which is an advanced form of mechanization. Mechanization's purpose is to relieve man of certain duties using human energy for power; automation's purpose is to relieve him of certain mental tasks and the related physical tasks necessary for their expression. Most of us sense that electronic computers, which are themselves examples of automation, will play an important role in the mechanization of control processes of the routine type.

It is generally conceded, to quote one authority,** that "higher level decisions will be made by man primarily because he, through the exercise of his mind, possesses the only means of integrating data for which rational formulations are not yet possible or are too expensive." And yet we are increasingly aware that even in the realm of higher order controls, particularly those mental tasks involving selection from among alternative courses of action, that this too is undergoing mechanization. To be precise, I am referring to the mental tasks known as programming (or scheduling) and their physical realization known as production control. Again we sense the important role that electronic computers will play.

* Presented March 2, 1956, before faculty colloquium, Univ Cal., Berkeley, Calif.
** "Some Reflections on Automation" by L.M.K. Boelter, Symposium on Electronics and Automatic Production, San Francisco, Aug, 1955.

All of us more or less associate these two kinds of postwar developments -- automation and programming -- possibly because they both make use of electronic computers. How closely are they related? Is it possible that the mechanization of certain of the more complex control processes are really the beginnings of an age of super-automation?

Will the logical steps in this development be:

1. Mechanization: Machines replace human energy tasks.
2. Automation: Machines replace simpler human control tasks.
3. Super Automation: Machines replace complex human control tasks.

With this possibility in mind, let us review developments in programming.

One of the reasons why the programming tool has assumed importance, both in industry and in the military establishment, is that it is a method for studying the behavior of systems. In philosophy it is close to what some describe as the distinguishing feature of management science or operations research, to wit: "Operations are considered as an entity. The subject matter studied is not the equipment used, nor the morale of the participants, nor the physical properties of the output, it is the combination of these in total as an economic process."*

To many the term "linear programming" refers to mathematical methods for solving linear inequality systems. While this may be the central mathematical problem it is not its definition. Linear Programming is a technique for building a model for describing the interrelations of the components of a system. As such it is probably the simplest mathematical model that can be constructed of any value for broad programming problems of industry and government.

* Operations Research for Management, C. C. Hermann and J. F. Magee, Harvard Bus. Rev., July, 1953.

Thus the importance of the linear programming model, is that it has wide applicability. The fact that a moderate size system can be mechanically solved, is the reason we say that mechanization has already made inroads into higher order control processes.

Having developed the role it may play in the trend towards automation, let us now proceed to our main task of describing its underlying philosophy and to point out areas where it has been successfully applied.

Suppose that the system under study (which may be one actually in existence or one which we wish to design) is a complex of machines, people, facilities, and supplies. It has certain overall reasons for its existence. For the military it may be to provide a striking force or for industry it may be to produce certain types of products.

The linear programming approach is to consider the entire system as decomposable into a number of elementary functions called "activities"; each type of activity is abstracted to be a kind of "black box" into which flow tangible things such as supply, money, and out of which may flow the products of manufacture or trained crews for the military. What goes on inside the "box" is the concern of the engineer or the educator, but to the programmer, only the rates of flow in and out are of interest.

The next step in building a model is to select some unit for measuring the quantity of each activity. For a production type activity it is natural to <u>measure</u> the quantity of the activity by the amount of some product produced by it. This quantity is called the <u>activity level</u>. To increase the activity level it will be necessary of course to increase the flows into and out of the activity. <u>In the linear programming model the quantities of flow of various</u>

items into and out of the activity are always proportional to the activity level. Thus it is only necessary to know the flows for the unit activity level. If we wish to double the activity level we simply double all the corresponding flows for the unit activity level.

While any positive multiple of an activity is possible, negative quantities of activities are not possible. The Mad Hatter, you may recall in Alice in Wonderland, was urging Alice to have some more tea, and Alice was objecting that she couldn't see how she could take more when she hadn't had any. "You mean, you don't see how you can take less tea," said the Hatter, "it is very easy to take more than nothing." Lewis Carrol's point, of course, is that the activity of "taking tea" cannot be done in negative quantity.

One of the items in our system is regarded as precious in the sense that total quantity of it produced by the system measures the payoff. The contribution of each activity to the total payoff is the amount of the precious item that flows into or out of each activity. Thus if the objective is to maximize profits, activities that require money contribute negatively and those that produce money contribute positively to total profits.

Next, it is required that the system of activities be complete in the sense that a complete accounting by activity can be made of each item. To be precise, for each item it is required that the total amount on hand equals the amount flowing into the various activities minus the amount flowing out. Thus, each item, in our abstract system, is characterized by a material balance equation -- the various terms of which represent the flows into or out of the various activities.

The programming problem consists in determining values for the activity levels which are positive or zero such that flows of each item (for these activity levels) satisfy the material balance equations and such that the value

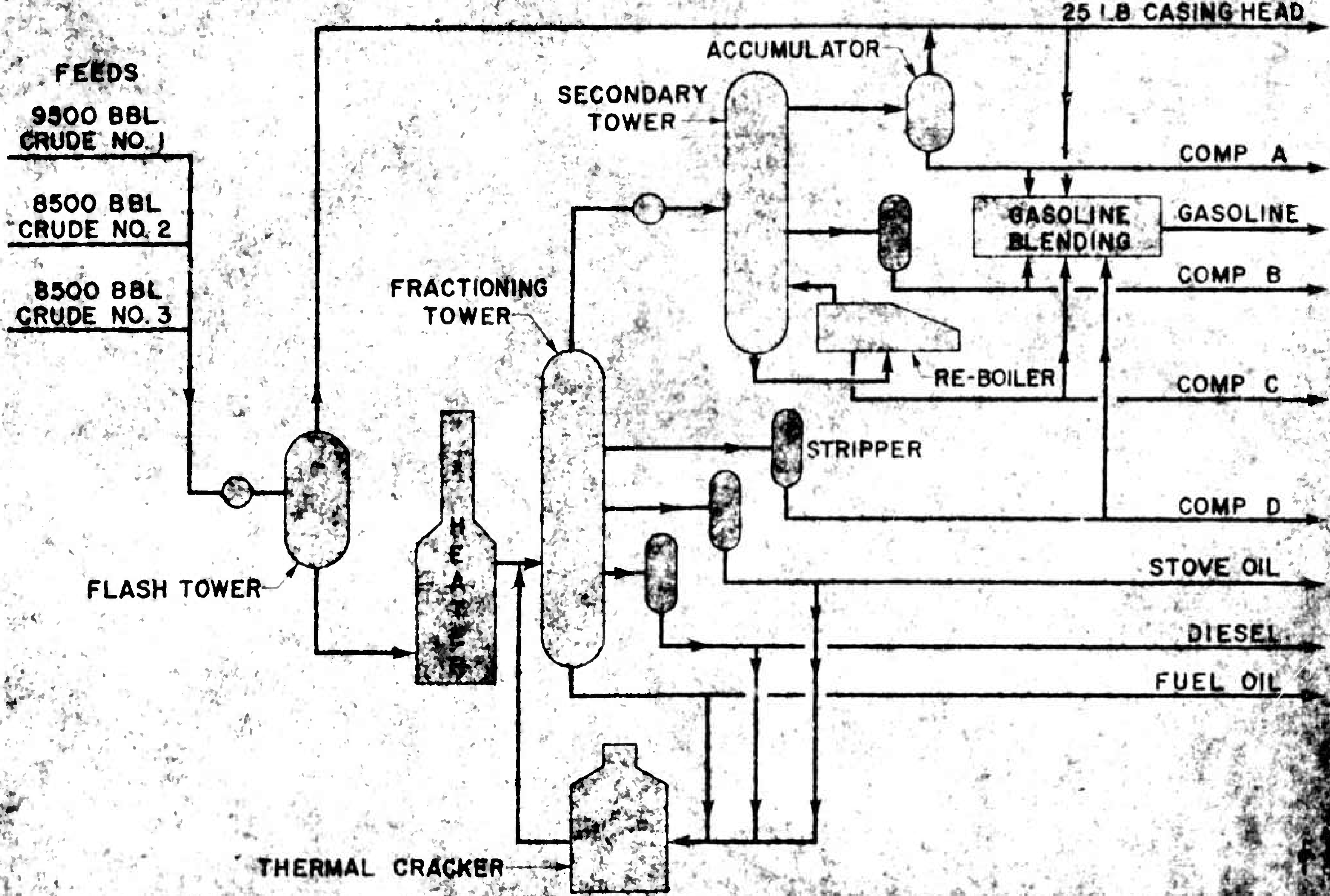
of the payoff is maximum. It is clear that what we have done is to reduce the programming problem to a well-defined mathematical problem which is called the LINEAR PROGRAMMING MODEL.

To illustrate these principles of the linear programming approach to model building, let us turn to an application in the petroleum industry where linear programming methods have been very successful. The complicated piece of plumbing of figure 1 is a flow diagram of one of the less complicated oil refineries.* The problem facing management is this. By turning valves, setting temperatures, pressures, and starting pumps, crude oil will be drawn from one or several oil fields under the control of the refinery (shown on the left). Like the old song about the music, it "will go around and around" and come out as several streams of pure oils (shown on the right). The latter can be marketed at varying prices. By changing the controls, the quantities in various streams of pure oils can be altered. This will change the costs of operating the equipment and the revenues from the sales of the final products. The various components are interrelated, however, in such a complicated manner, that it is not obvious what is the <u>best way</u> to operate the equipment to <u>maximize profits</u>. In spite of these complex interrelations, when this system is decomposed into elementary functions as the first step in building a model, it turns out that there are essentially only three main kinds of activities taking place: Distillation, Cracking, Blending.

<u>Distillation Activity:</u> The net effect of the flash tower, heater, fractionating towers, strippers, etc. is to separate the crude into varying amounts of pure oils of which it is composed. Crudes drawn from different oil fields will have different decompositions. Hence there must be separate distillation activity developed for each type crude. The maximum amount of crude that

* Refinery example taken from a term paper of R. J. Ullman.

REFINERY FLOW DIAGRAM

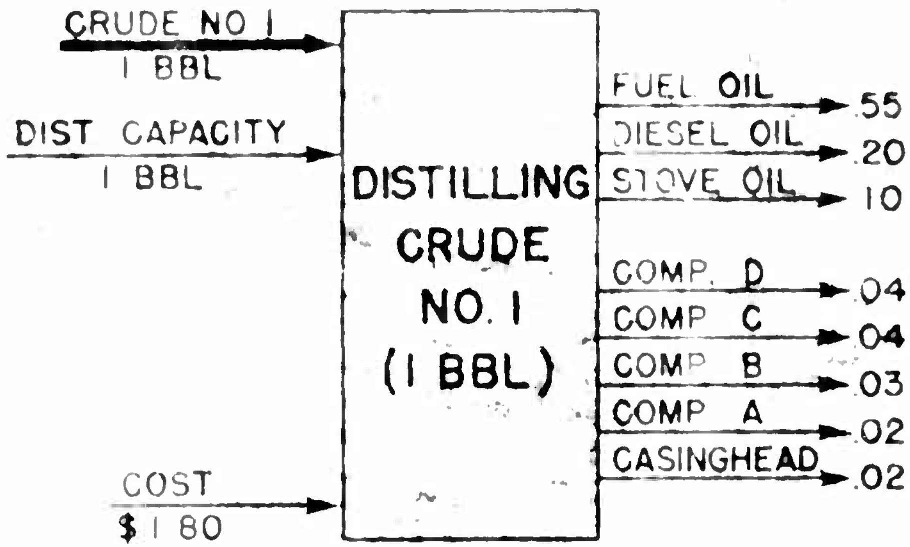


- fig. 1 -

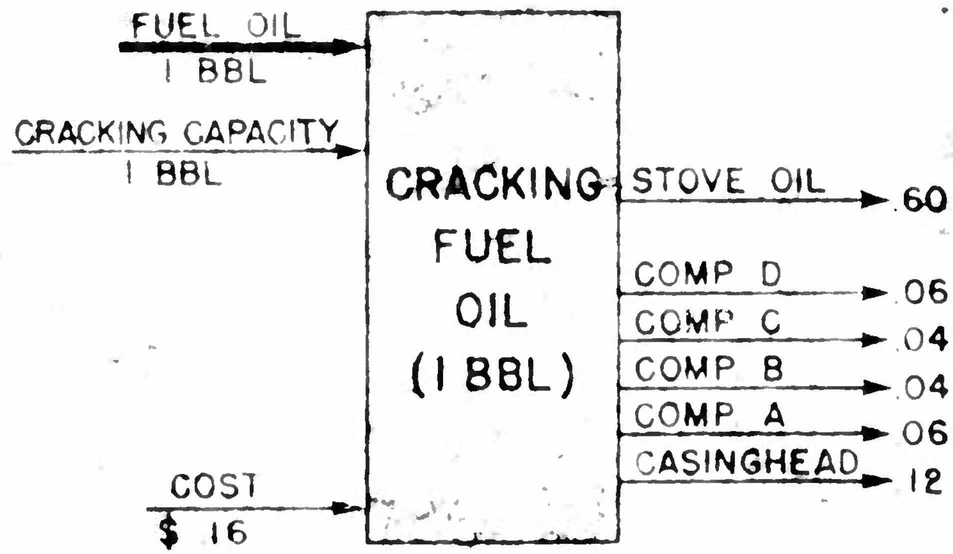
can be distilled depends on which, of the varying pieces of equipment it passes through, will be the bottleneck. In our case we will suppose it is the heater and that it has a fixed capacity of 14,000 bbl's per day independent of type crude processed. From this description it is evident if the level of distallation activity is measured in number of barrels of crude input, then a unit level of activity can be pictured as in Figure 2A. It is seen that 1 bbl of Crude No. 1 will use 1 bbl of distillation capacity, and will cost $1.80 (to purchase and to distill), the outputs will be a stream of pure oils in the amounts shown. These outputs are principally the heavier oils: fuel, diesel and stove and smaller amounts of the lighter types used to make gasoline. If instead of 1 bbl, it is desired to distill 10 or X bbls of crude all input and output quantities of Figure 2A would have to be multiplied by 10 or X.

<u>Cracking:</u> The net effect of the cracking equipment is to take one of the heavier type oils and to cause it to be broken down into lighter type oils. In the case of fuel oil it will produce a small amount of the lighter types and a larger amount of stove oil which, if desired, can in turn be recycled back into the cracker and made into lighter oils. It is seen from Figure 2B that 1 unit of fuel oil requires 1 unit of cracking capacity, will cost $ .16 and will produce the pure oils in the amounts shown on the right. A separate type activity must be set up for cracking: fuel, diesel and stove oils.

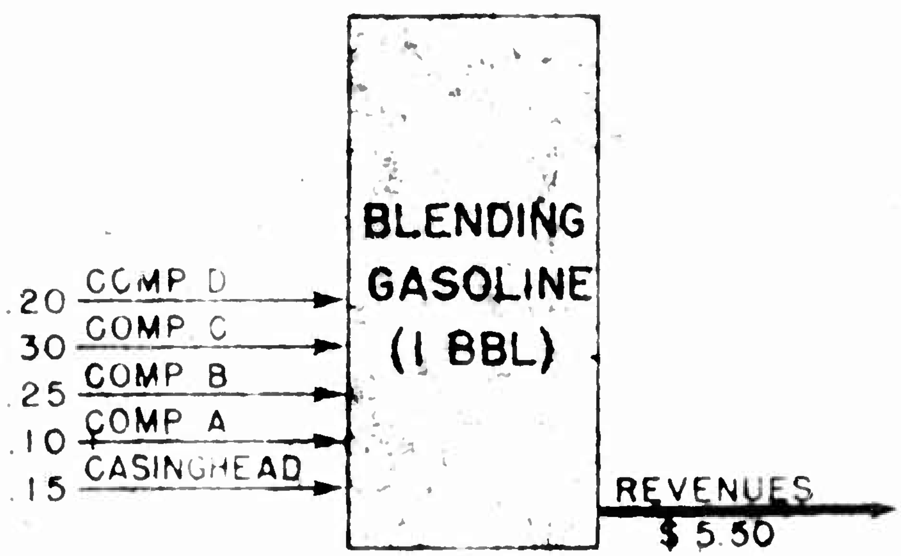
<u>Blending:</u> Gasoline is not a pure oil but is a blend of several of the lighter types of pure oil (see Figure 2C). It will be noted the only output shown is the <u>net revenues</u> from marketing 1 bbl of gasoline. The latter is assumed to be the sales price at the refinery less the cost of the



(fig. 2a)



(fig. 2b)



(fig. 2c)

-fig. 2-

blending operation.

Once the flows for these major activities have been determined on per-barrel basis, it is a simple matter to set up the linear programming model by means of which the managers can determine the best manner to operate the refinery to maximize profits. In Figure 3 each column represents an activity. The input and output quantities per unit level of activity is shown in the column; to distinguish outputs from inputs, outputs are shown with a minus sign. For example, the data of Figure 2A is shown in column "Distallation - Crude 1"; the data of Figure 2B is shown in column "Cracking - Fuel Oil"; the data of Figure 2C is shown in column "Product Marketing-Gasoline". The other activity columns are self-explanatory. The amounts available of various items to the system, are shown on the right.

The unknown activity levels to be determined are denoted by $x_1, x_2, \ldots, x_{20}$. By multiplying these unknowns by the corresponding numbers found in any row and summing the terms across, the total obtained should equal the availability shown on the right.

For example, the first material balance equation reads

$$1.x_1 + 1.x_4 = 9500.$$

which means the amount of crude No.1 available, 9500 bbls, is completely accounted for by the amount left in the ground x_1 plus the amount distilled x_4.

The fourth material balance equation, referring to the item distillation capacity, reads simply

$$1.x_4 + 1.x_5 + 1.x_6 + 1.x_7 = 14{,}000$$

which means that 14,000 barrels of distillation capacity is completely accounted for by the amount used in distilling the various types of crudes plus any excess capacity not used.

LINEAR PROGRAMMING MODEL OF A REFINERY

ACTIVITIES / ITEMS	UNUSED			Distillation			Unused Crude Capacity	CRACKING			Unused Crack Capacity	PRODUCT MARKETING									AVAILABLE (BBLS/DAY)
	Crude 1	Crude 2	Crude 3	Crude 1	Crude 2	Crude 3		Fuel Oil	Diesel Oil	Stove Oil		Fuel Oil	Diesel Oil	Stove Oil	Gasoline	Comp. D	Comp. C	Comp. B	Comp. A	Casinghead	
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	
Crude 1	1			1																	= 9500
Crude 2		1			1																= 8500
Crude 3			1			1															= 8000
Crude Capac.				1	1	1	1														= 14,000
Fuel Oil				-.55	-.61	-.50		1				1									= 0
Diesel Oil				-.20	-.12	-.11			1				1								= 0
Stove Oil				-.10	-.07	-.14		-.6	-.2	1				1							= 0
Crack. Capac.								1	1	1	1										= 3,500
Comp. D				-.04	-.06	-.05		-.06	-.41	-.30					.20	1					= 0
" C				-.04	-.05	-.08		-.04	-.20	-.30					.30		1				= 0
" B				-.03	-.04	-.05		-.04	-.04	[illegible]					.25			1			= 0
" A				-.02	-.02	-.03		-.06	-.12	-.10					.10				1		= 0
Casinghead				-.02	-.03	-.04		-.12	-.16	-.14					.15					1	= 0
Profit				-1.8	-1.2	-2.0		-.16	-.21	-.21		1.8	4.0	4.2	5.5	4.0	4.1	4.2	4.3	3.3	= MAXIMUM

-fig. 3-

Finally the profit equation states the revenues obtained from marketing various products, $(1.8x_{12} + 4.0x_{13} + 4.2x_{14} + 5.5x_{15} + 4.3x_{16} + 4.1x_{17} + 4.2x_{18} + 4.3x_{19} + 3.3x_{20})$, less the cost of distilling and crude purchases, $(1.8x_4 + 1.9x_5 + 2.0x_6)$, less the cost of cracking, $(.16x_8 + .21x_9 + .21x_{10})$, is the amount of profit. The problem of course is to choose the program of activity levels in such a way that the material balance equations are satisfied and the profits maximized.

— — —

Starting with the Gulf Oil Company a few years ago, linear programming is now used by nearly every large producer in the petroleum industry. In California, Standard Oil, Shell, Richfield, and Union are all active. What kind of money do they expect to make with it?

Those who schedule refinery operations are very experienced and with or without linear programming do a very fine job. It is doubtful that the new tool (as it is currently applied) helps them increase the gross (adjusted for changes in operating costs) by much more than 1%. This does not sound like much but it should be remembered that major oil companies in California do anywhere from roughly a million to several million dollars business per day. Using the figure of a million dollars, the increase in profits can be around $10,000 a day or 3 & 1/2 million dollars a year. Since oil companies make about 15% profit on gross before taxes and reinvestment, this can mean to them at least 5% increase in their profits.

From this example, it can be inferred that linear programming models have had a large number of applications both military and industrial. The growth of industrial uses during the past few years is quite remarkable. Next to the

petroleum industry, the feed industry is perhaps the second most active customer. Heinz Company uses it to determine which of a half dozen plants should ship ketchup to 70 warehouses located in all parts of the country. A major milk producer is currently developing it as a means of determining which of its dozen or so milk producing centers will ship canned milk to thousands of warehouses in this country and to foreign markets. Its transportation bill, incidentally, runs in the millions each month. Armour and Company use it to determine the most economical mixture of feeds for animals.

In metal processing applications, Argus Cameras reports a savings of $54,000 per quarter in its screw machine department because the price implications of the linear programming approach helped them to decide whether it paid to make or to buy a part. SKF prior to 1953 used a scheduling method well known to business called a Gantt Control System to decide which job should be done on which equipment. Thus in their screw machine area there were 120 machines varying from one inch multiple spindle to eight inch single spindle. Any given job can be done on as many as five different sizes and makes of equipment. Dry-run comparisons between the Gantt chart system and the Linear Programming Solution indicated that 6000 machine hours were wasted each month. They estimate by use of linear programming that they have made an equivalent of a 15-man reduction or about $100,000 per year savings.

Let us turn our attention to computation. Oddly enough, prior to 1947 there existed no systematic procedures for solving linear programming problems. Accordingly, all that is known about the computation of such systems has developed in less than nine years.

One of the characteristics of the linear programming models is that they usually involve a large number of items and a still larger number of activities. Thus the Heinz Ketchup example results in a mathematical system involving 76 equations in 420 unknowns; the major milk producer example results in a system of more than 1000 equations and 10,000 unknowns. Here is a military application that results in a system of 10,000 equations in 50,000 unknowns: The Air Force tests the abilities of as many as 10,000 inductees at a time as to their abilities in 5 job classifications and, on this basis, tries to assign them according to ability and the number required in each category of job. They are currently considering linear programming to help them find a better overall assignment.

Fortunately, these problems belong to the special class referred to as "transportation and assignment" problems for which remarkably efficient procedures have been developed after much research. Without these special procedures the last two examples would be impossible to consider.

As a rule, the size of the model is not dictated by the application but is one that has been adjusted downward to fit current computing capabilities. For example, RAND has computed a number of problems for General Electric Plant at Hanford (which, incidentally, uses linear programming to plan their operations). Their models ran characteristically 150 equations because this was the practical limit of our computation capability. Incidentally, RAND's electronic computer instruction code is considered as a remarkable achievement because it can handle with ease a one-hundred-fifty equation system. Because of this, other computation centers have been copying its features.

As the use of the linear programming model for systems studies becomes more accepted, it is axiomatic that there will be greater and greater pressure

to apply it to larger and larger systems.

The greatest promise of rapid solution of the larger systems does not appear to depend on the development of bigger and faster computers and better instruction codes (although this will help) but rather upon the classification of model structures and the development of special methods for each class. This is the reason why the mathematical research program at RAND has concentrated on methods for solving special types of linear programming systems. The surface has hardly been scratched but here is an example of what has been accomplished already.

1. The "transportation" model referred to earlier represents a class which has had perhaps more important applications than any other. As a result of research at RAND and Princeton, a remarkably efficient method has recently been developed that permits rapid solution for this class; even for the case of hundreds of equations in thousands of unknowns. It is work of this type that makes it possible to solve a problem as large as the Air Force personnel assignment example.

Let us return again to the relation of linear programming to automation by considering an industry that has been one of the first to automatize - the petroleum industry. Indeed, as we have seen the interrelationships within a modern refinery are so complex that it is no longer possible to properly evaluate a given operational scheme without such a tool. In a word, there are just too many combinations of feed stocks, operating sequences, operating conditions, ways to blend, and choices of final products.

This is a good area to illustrate why programming constitutes a higher order control. It is not a feedback device for holding a boiler at a fixed

temperature or pressure but a method for deciding what the temperature or pressure settings should be and for how long. Since different units of the refinery run for long periods before it is necessary to change settings of the controls, the program is usually set up each month or quarter of year and is translated by the engineers into day-to-day operations. Thus it appears that even in a highly automatised industry it is not necessary that the higher order controls be completely mechanized. Indeed, here it is worthwhile to mechanize the mental tasks and not the accompanying physical tasks. For the most part the latter take the form of written orders to various echelons of management finally terminating with the setting of the lower order control by hand.

It is possible, however, that as automation becomes more extensively employed in certain industries that the higher ordered d cisions will be more mechanically linked to the lower ordered decisions. For example, the aircraft industry uses small numbers of a great variety of parts. For this application automation appears to be taking the form of flexible single machines that can produce a great variety of products. The plan is to have the machines fed detailed instructions by hooking them up to electronic computers via tapes and cards. In such a system it may be necessary to reprogram the workload of machines frequently, particularly if there is a policy of short lead times on orders. Another reason for rapid reprogramming would be a machine breakdown. Thus this application will probably require both tight methods of production control and flexible programming techniques.

To conclude our remarks on automation, we note again that linear programming makes it possible to mechanize part of the higher order controls. It appears

likely that as this mechanization becomes more accepted, it will be linked more and more to tighter production control. In a word, automation may become super-automation.